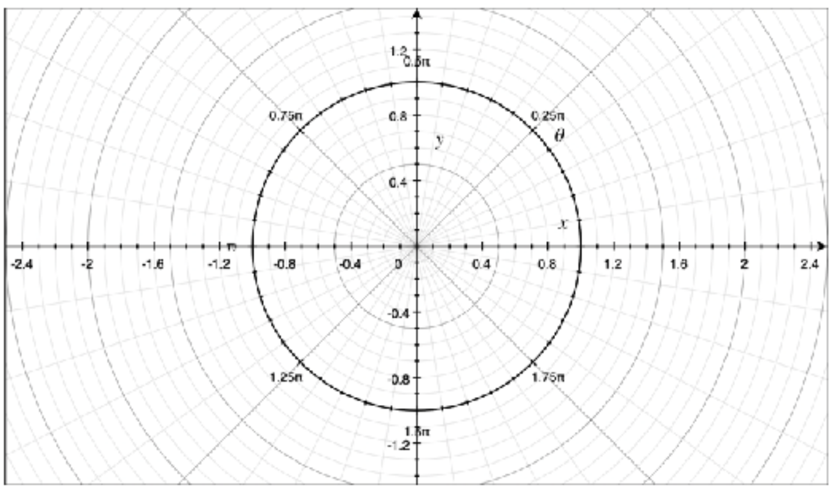
8.1 Polar Coordinates

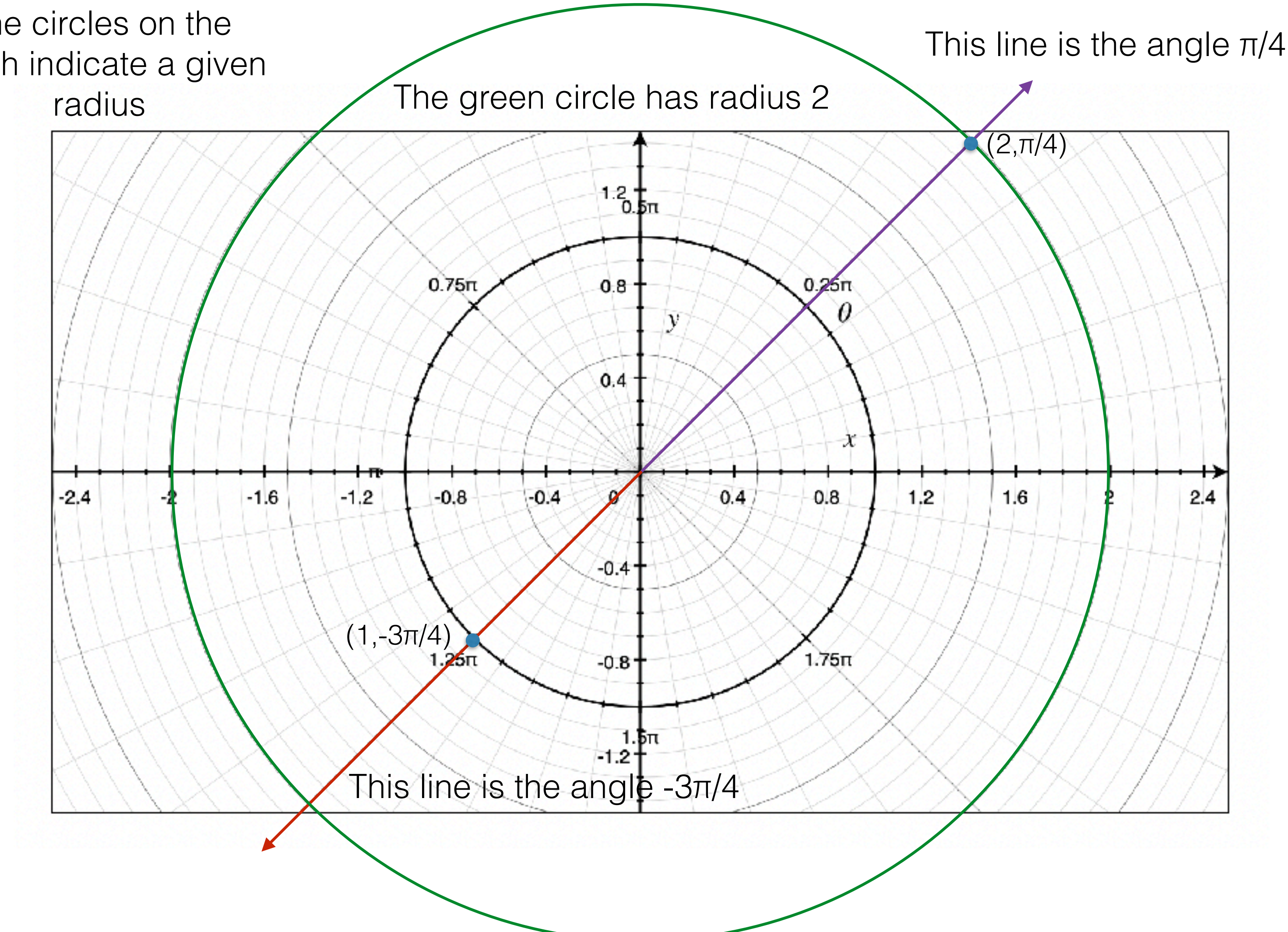
<u>Objectives</u>: 1) Plot points using polar coordinates
2) Convert from Polar coordinates to rectangular coordinates
3) Convert from Rectangular coordinates to Polar coordinates
4) Transform Equations between Polar and rectangular Forms



Polar Coordinates are an extension of working with the unit circle. We can define points by a radius, r and a direction, θ

Graph the points (2,π/4) and (1,-3π/4)

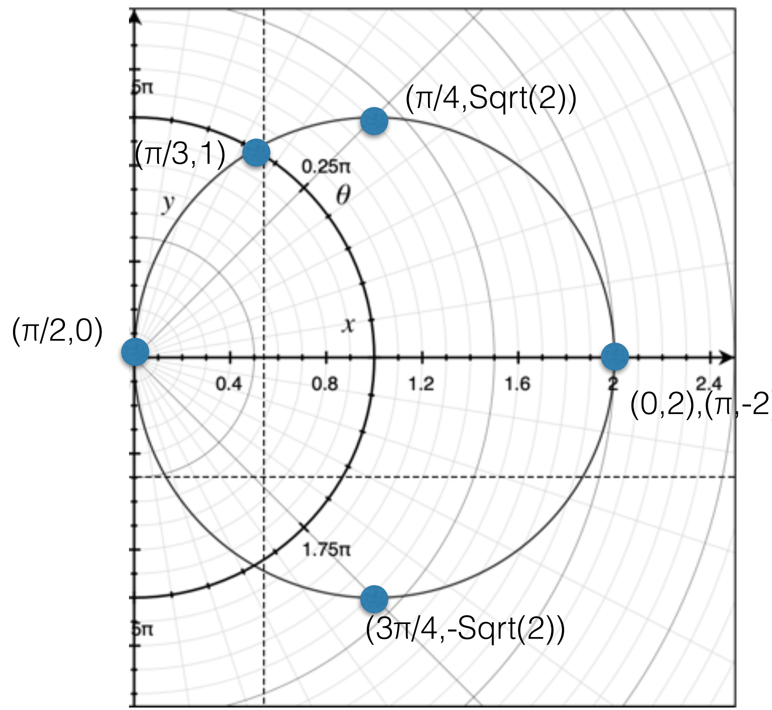
The circles on the graph indicate a given radius



Graphing Equations

We can use the technique of graphing points to graph polar equations. Let's graph r=2cos(θ). The easiest way is to make a data table of points with values of θ that we know. For example 0, π/4, π/2, etc

θ	R
0	2
π/4	Sqrt(2)
π/3	1
π/2	0
3π/4	-Sqrt(2)
π	-2



Converting from Polar to rectangular:

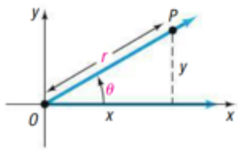


Figure 12

To convert from polar to rectangular we want to write (r,θ) as (x,y) instead. The trigonometry we've done already in this course gives us a way. Note that sin(θ)=y/r, so y= r sin(θ). Since we know what r and θ are we can solve for y. Similarly we can get x=r cos(θ)

Examples: Convert the following from polar coordinates (r,θ) to rectangular coordinates (x,y).

(5, π/4)

$$x = 5\cos(\pi/4)$$
$$x = \frac{5\sqrt{2}}{2}$$
$$y = 5\sin(\pi/4)$$
$$y = \frac{5\sqrt{2}}{2}$$
$$\left(\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right)$$

(6, 5π/6)

$$x = 6\cos(5\pi/6)$$
$$x = \frac{-6\sqrt{3}}{2}$$
$$y = 6\sin(5\pi/6)$$
$$y = \frac{6\cdot 1}{2}$$
$$\left(\frac{-6\sqrt{3}}{2}, 3\right)$$

Converting from Rectangular to polar:

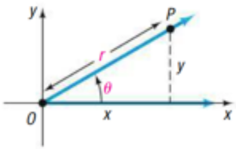


Figure 12

To convert from rectangular to polar we want to write (x,y) as (r,θ) instead. The trigonometry we've done already in this course gives us a way. Note that $r^2=x^2+y^2$ from the pythagorean theorem. Since we know what x and y are we can solve for r. Additionally we have that tan(θ)=y/x. So, θ=arctan(y/x) adjusting for the right quadrant

Steps for Converting from Rectangular to Polar Coordinates

STEP 1: Always plot the point (x, y) first, as shown in Examples 5, 6, and 7. Note the quadrant the point lies in or the coordinate axis the point lies on.

STEP 2: If $x = 0$ or $y = 0$, use your illustration to find r.
If $x \neq 0$ and $y \neq 0$, then $r = \sqrt{x^2 + y^2}$.

STEP 3: Find θ. If $x = 0$ or $y = 0$, use your illustration to find θ.
If $x \neq 0$ and $y \neq 0$, note the quadrant in which the point lies.

$$\text{Quadrant I or IV:} \quad \theta = \tan^{-1}\frac{y}{x}$$

$$\text{Quadrant II or III:} \quad \theta = \pi + \tan^{-1}\frac{y}{x}$$

Examples: Convert from rectangular coordinates to polar coordinates

$(-4, 4\sqrt{3}) \longrightarrow$ (8,2π/3)

$$r^2 = (-4)^2 + (4\sqrt{3})^2$$
$$= 16 + 16 \cdot 3$$
$$= 64$$
$$r = 8$$
$$\tan(\theta) = \frac{4\sqrt{3}}{-4}$$
$$= -\sqrt{3}$$
$$\theta = \frac{2\pi}{3}$$

This is because the point is in the II quadrant

$(0, -3) \longrightarrow$ (3, 3π/2)

This is on the negative y-axis

$(-3, -4) \longrightarrow$ (5,4.069)

$$r^2 = 3^2 + 4^2 = 25$$
$$r = 5$$
$$\tan(\theta) = \frac{-4}{-3}$$

Notice that (-3,-4) is in the third quadrant so

$$\theta = \pi + \arctan(\frac{4}{3}) = 4.06889$$

4.

Which of the following are the same as the point $(-3, \frac{4\pi}{3})$? (Hint: there are three answers)

A. $(-3, -\frac{2\pi}{3})$

B. $(-3, \frac{\pi}{3})$

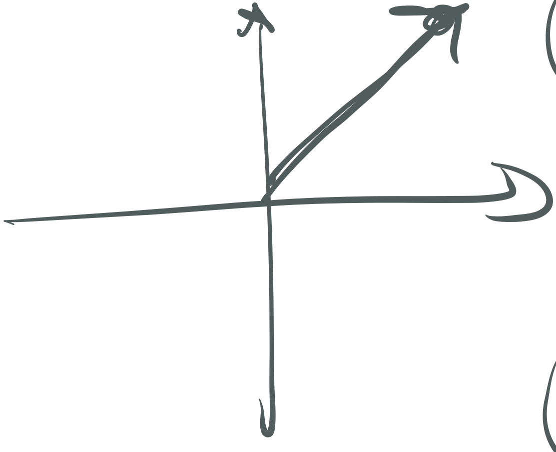
C. $(-3, \frac{2\pi}{3})$

D. $(3, -\frac{5\pi}{3})$

E. $(3, -\frac{4\pi}{3})$

F. $(3, \frac{\pi}{3})$

G. $(3, \frac{4\pi}{3})$



$(-3, -\frac{2\pi}{3})$

$(3, \frac{\pi}{3})$

$(3, -\frac{5\pi}{3})$

5.

Convert $(2, \pi/3)$ from polar coordinates to rectangular coordinates.

$x = 2\cos(\pi/3) = 2\left(\frac{1}{2}\right) = 1$

$y = 2\sin(\pi/3) = 2\left(\sqrt{3}/2\right) = \sqrt{3}$

6.

Convert $(1, -\sqrt{3})$ from rectangular coordinates to polar coordiates.

$$r^2 = 1^2 + (-\sqrt{3})^2 = 4$$

$$r = 2$$

$$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$$

$$\left(2, -\frac{\pi}{3}\right)$$

7.

Convert (-3,-4) from rectangular coordinates to polar coordinates.

$r^2 = (-3)^2 + (-4)^2 = 25$

$r = 5$

Calculator

$\theta = \tan^{-1}\left(\frac{-4}{-3}\right) = 0.927295$ but

(-3,-4) is in the third quadrant not first.

$(r, \theta) = (5, 0.927295 + \pi)$

8.

Convert (0,-3) from rectangular coordinates to polar coordinates.

$(3, \pi)$

9.

Convert (-4, $4\sqrt{3}$) from rectangular coordinates to polar coordinates.

$= 8(-1/2, \sqrt{3}/2)$

$\left(8, \frac{2\pi}{3}\right)$

10.

Convert (6,5π/6) from polar coordinates to rectangular coordinates.

$$x = 6\cos(5\pi/6) = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$$

$$y = 6\sin(5\pi/6) = 6(1/2) = 3$$

11.

Convert (5,π/4) from polar coordinates to rectangular coordinates.

$$x = 5\cos(\pi/4) = \frac{5\sqrt{2}}{2}$$

$$y = 5\sin(\pi/4) = \frac{5\sqrt{2}}{2}$$

12.

Tranform the polar equation to an equation in rectangular coordinates.

$$r = 2\sin(\theta)$$

A. $x^2 + (y-1)^2 = 1$

B. $(x-1)^2 + y^2 = 1$

C. $x = 2$

D. $y = 2$

circle with center $(0,1)$ radius 1.

$$r^2 = 2r\sin\theta = 2y$$

$$x^2 + y^2 = 2y$$

$$x^2 + (y^2 - 2y + 1) = 1$$

$$x^2 + (y-1)^2 = 1$$

13.

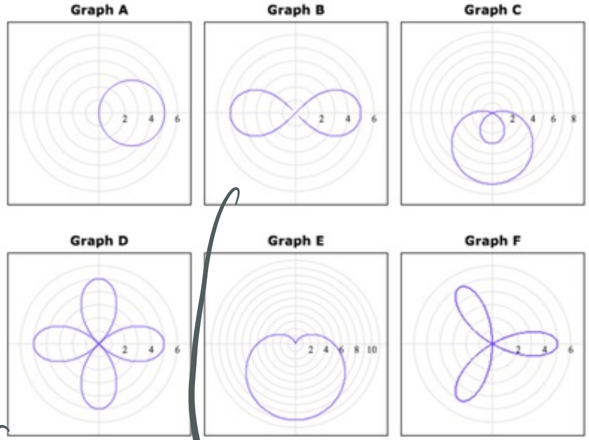
Transform the following polar equation to an equation in rectangular coordinates.

$$r\cos(\theta) = 4$$

$x = r\cos(\theta)$ so $x = 4$

15.

Match the polar equations with the graphs. See how many you can guess without using graphing software, then use graphing software to help.



$r = 5 - 8\sin\theta$

$r = 5\cos 3\theta$

A. Graph A 1. $r = 5\cos\theta$

B. Graph B 2. $r^2 = 5\cos 2$

C. Graph C 3. $r = 5 - 8\sin\theta$

D. Graph D 4. $r = 5\cos 2\theta$

E. Graph E 5. $r = 5 - 5\sin\theta$

F. Graph F 6. $r = 5\cos 3\theta$

$r = 5\cos\theta$

$r^2 = 5\cos 2\theta$

$r = 5\cos 2\theta$

$r = 5 - 5\sin\theta$

plug in values of θ and plot.