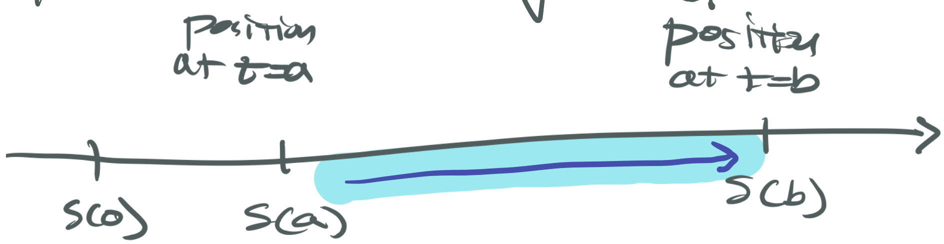
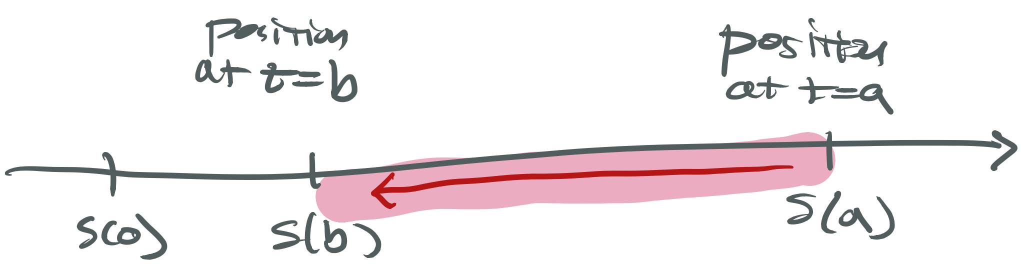
Suppose you're driving along a straight highway and your position relative to a reference point is $s(t)$ for times $t \geq 0$. Your displacement over a time interval $[a, b]$ is the change in position $s(b) - s(a)$.

If $s(b) \geq s(a)$, then your displacement is positive, when $s(b) < s(a)$, your displacement is negative.



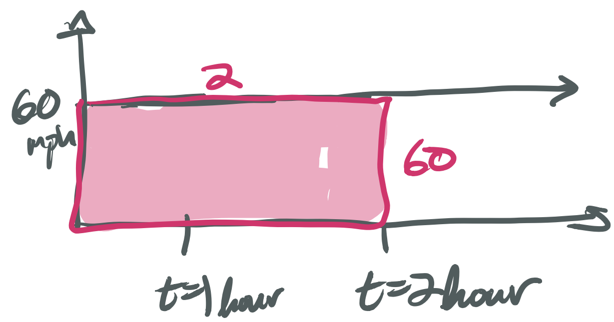
Displacement $= s(b) - s(a) \geq 0$



Displacement $= s(b) - s(a) < 0$

Now assume that $v(t)$ is the velocity of the object at a specific time.

<u>Example</u>: Suppose you drive at a constant speed of 60mph. Then $v(t) = 60$.



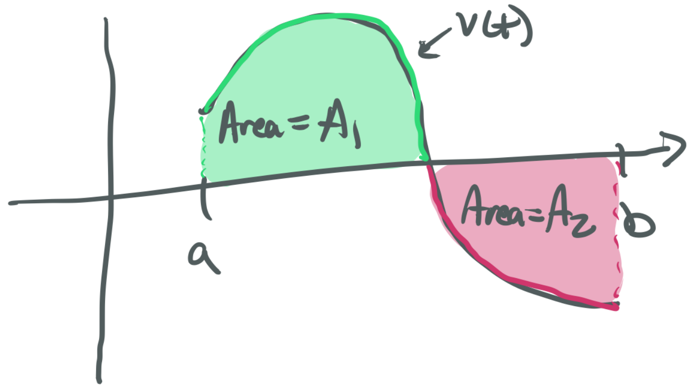
If you want to find distance traveled you just calculate $60 \cdot t$. But this is the same as the area under the curve. For example. After 2 hours you travel 120 miles which is the area of Red Rectangle.

Now we can extend this to general velocity functions $v(t)$. Let $s(t)$ be the antiderivative of $v(t)$. I.e. $s'(t) = v(t)$. Then the displacement from time $t=a$ to time $t=b$ is given by

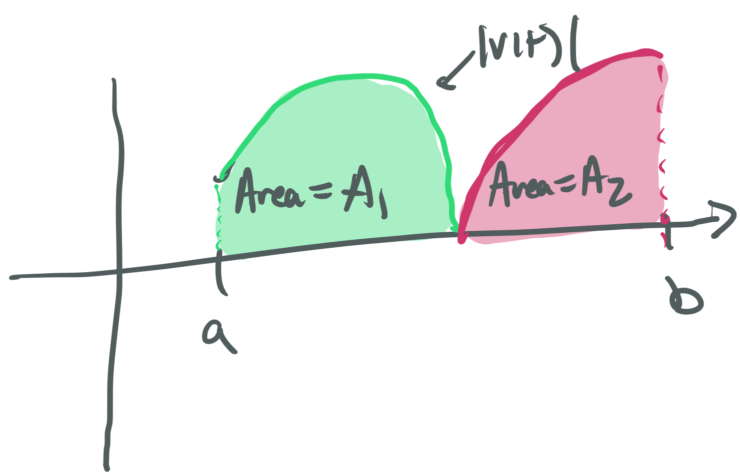
$$\int_a^b v(t)\,dt = \int_a^b s'(t)\,dt = s(b) - s(a).$$

Note: Distance traveled is given by $\int_a^b |v(t)|\,dt$. It is <u>different</u> from Displacement.

Example:

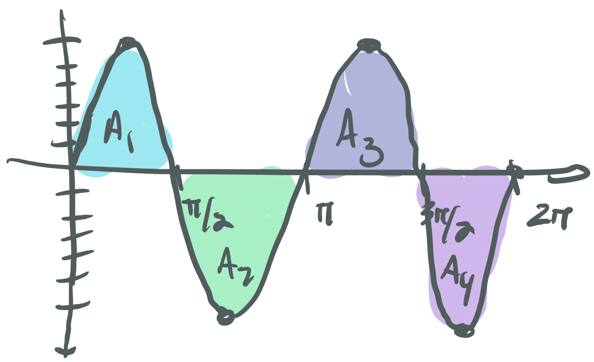


$$\text{Displacement} = A_1 - A_2 = \int_a^b v(t)\,dt$$



$$\text{Distance traveled} = A_1 + A_2 = \int_a^b |v(t)|\,dt$$

Example: Suppose an object moves along a line with velocity $V(t) = 6\sin(2t)$ for $0 \le t \le 2\pi$, where t is measured in seconds.



a) Find the distance traveled by the object on the time interval $[0, \pi/2]$

$$\text{distance traveled} = \int_0^{\pi/2} 6\sin(2t)\,dt$$

$$= -3\cos(2t)\Big|_0^{\pi/2} = -3\cos(\pi) - (-3\cos(0))$$

$$= 3 - (-3) = 6.$$

b) Find the distance traveled by the object on the time interval $[0, 2\pi]$.

$$A_2 = \int_{\pi/2}^{\pi} 6\sin(2t) = -3\cos(2t)\Big|_{\pi/2}^{\pi}$$

$$= -3\cos(2\pi) - (-3\cos(\pi)) = -3 - 3 = -6.$$

Continuing we can find $A_3 = 6$ $A_4 = -6$

So the total distance is

$$|A_1| + |A_2| + |A_3| + |A_4| = 6+6+6+6 = 24.$$

However, notice displacement $= A_1 + A_2 + A_3 + A_4$
$= 0.$

We can extend this idea to acceleration

Theorem 6.2

Given the acceleration $a(t)$ of an object moving along a line an its initial velocity $v(0)$, the velocity of the object for future times $t \geq 0$ is

$$v(t) = v(0) + \int_0^t a(x)\,dx.$$

Example:

A ball is thrown straight up with initial velocity of 40 meters per second. Assume only the force of gravity acts on the ball with an acceleration of 9.8 m/s.

Find the velocity of the ball.

$$V(t) = V(0) + \int_0^t a(x)\,dx$$
$$= 40 + \int_0^t -9.8\,dx$$
$$= 40 + -9.8x\Big|_0^t = \boxed{40 - 9.8t}$$

<u>Note</u>: We can extend this idea other situations.

<u>Example</u>:

Suppose the amount of water that passes through the Hoover dam is given by $g(t) = 60 + 12\cos\left(\frac{\pi}{12}t\right)$

$g(t)$ is millions of gallons per hour. How much water passes through the Hoover dam in 1 day?

$$\#\,\text{Gallons} = \int_0^{24} 60 + 12\cos\left(\tfrac{\pi}{12}t\right)dt$$

$$= 60t + \frac{144}{\pi}\sin\left(\frac{\pi}{12}t\right)\Big|_0^{24}$$

$$= 60(24) + \frac{144}{\pi}\sin(2\pi)$$

$$- 60(0) + \frac{144}{\pi}\sin(0)$$

$$= \boxed{1440 \text{ million gallons}}$$