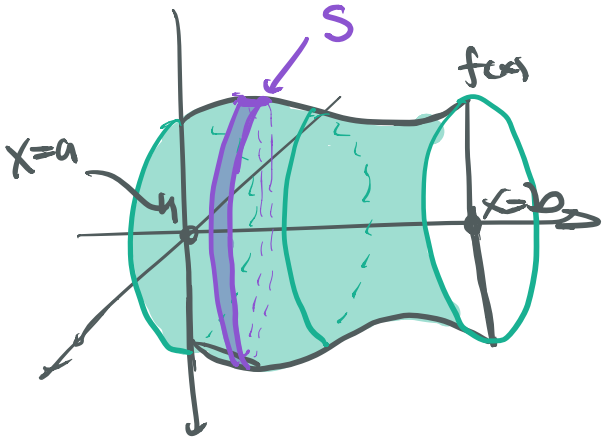
6.6 Surface Area

Surface area provides an extension to arclength. Let's consider a surface of revolution and try to find the surface area.



The surface area of S is roughly that of of that of a cylinder with radius $f(x)$ and height a small piece of the arclength.

$$S \approx 2\pi f(x)\,\Delta l \approx 2\pi f(x)\sqrt{1+f'(x)}\,\Delta x.$$

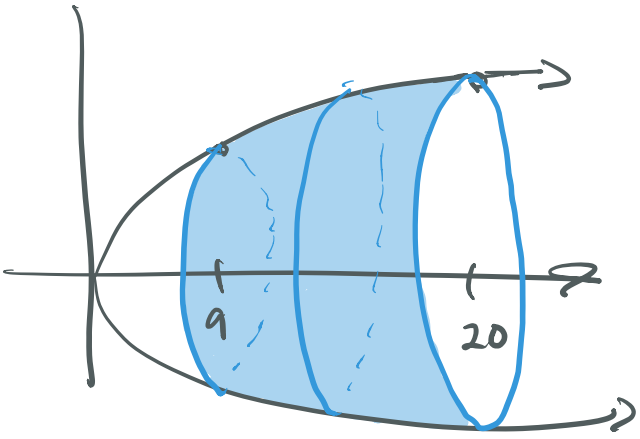
Δl: This is a small change in arclength.

Summing everything gives that

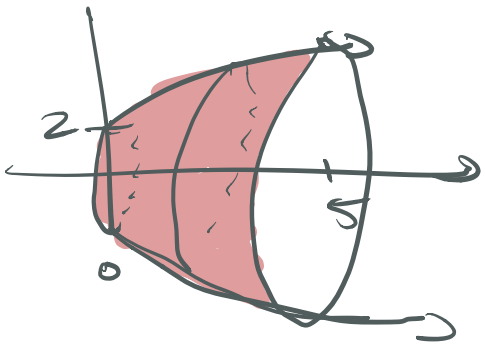
$$SA = \int_a^b 2\pi f(x)\sqrt{1+f'(x)^2}\,dx$$

Remark: There is a more formal derivation of this formula in the textbook.

Example: Find the area of the surface generated when $y = 8\sqrt{x}$, for $9 \le x \le 20$ about the x-axis.



Example: Find the surface area of $y = \sqrt{4x+6}$ rotated about the x-axis with $0 \le x \le 5$.



Example: Let $f(x) = \frac{1}{x}$. a) Find the volume of the region bounded by $f(x)$, x-axis, $x=1$ and $x=\infty$. rotated about the x-axis.

b) Show that the SA is ∞.