

Scalar valued functions of several variables

June 1, 2020

Big Picture:

- 1) A function of several variables maps a “height” above a point in the plane.
- 2) Level sets help us visualize a graph in three dimensions.

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Remark: If $z = f(x, y)$, think of $f(x, y)$ as a “height”. Then the domain is a set in \mathbb{R}^2 , and the range is the set of heights that f takes. The domain is in \mathbb{R}^2 , but the range is in \mathbb{R} .

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